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Holography and the electroweak phase transition

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Abstract: We study through holography the compact Randall-Sundrum (RS) model at finite temperature. In the presence of radius stabilization, the system is described at low enough temperature by the RS solution. At high temperature it is described by the AdS-Schwarzschild solution with an event horizon replacing the TeV brane. We calculate the transition temperature T_c between the two phases and we find it to be somewhat smaller than the TeV scale. Assuming that the Universe starts out at $T \gg T_c$ and cools down by expansion, we study the rate of the transition to the RS phase. We find that the transition is very slow so that an inflationary phase at the weak scale begins. The subsequent evolution depends on the stabilization mechanism: in the simplest Goldberger-Wise case inflation goes on forever unless tight bounds are satisfied by the model parameters; in slightly less-minimal cases these bounds may be relaxed.

Keywords: Extra Large Dimensions, AdS-CFT Correspondence, Black Holes, Cosmological Phase Transitions.

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Contents

1. Introduction

In recent years particle physics has been shaken by the realization that new compact space dimensions could be so large to be accessible in collider experiments [1]–[4]. This realization allows for a new viewpoint on the open problems of particle physics and most notably on the hierarchy problem. In particular in the scenario proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) [2] the fundamental scale of quantum gravity is of the order of the weak scale. A crucial feature of this proposal is that the Standard Model degrees of freedom should be localized on a 3-dimensional defect, a *brane*, so that they cannot access directly the new dimensions. Then the weakness of 4D gravity originates from the large radius R of compactification: solving the hierarchy problem would now consist in explaining why $1/R$ is so much smaller than the weak scale. Randall and Sundrum (RS) [4] have instead proposed a warped compactification with one extra dimension. The full space-time is a slice of AdS_5 bordered by two branes. The interest of this model is in that it can elegantly explain the hierarchy as due to gravitational red-shift of energy scales. Also in this case the hierarchy depends on the size of the extra dimension, but Goldberger and Wise (GW) [5] have shown a simple mechanism that can naturally generate the right order of magnitude.

The presence of new space dimensions dramatically changes the early evolution of our Universe. When the temperature is higher than $1/R$, Kaluza-Klein (KK) particles can be produced with important cosmological consequences, not always pleasant. Indeed in the ADD scenario the KK gravitons produced at high temperature tend to over-close the Universe and, upon decaying on the brane, to distort the diffuse photon spectrum. This places a rather strong bound of about $1-100\,\mathrm{MeV}$ on the maximal temperature at which the Universe was ever heated [2]. With such a low maximal temperature the standard Big Bang Nucleosynthesis barely fits into the picture. It is regarded as rather unnatural that standard cosmology should start right at the time of nucleosynthesis. In other models, like RS or ref. [1], the maximal temperature at which the Universe is surely normal is about $1\,\mathrm{TeV}$. This scale represents the separation of KK levels, and anyway it is about the scale where the interactions of these extra-dimensional theories go out of perturbative control and the effective field theory description breaks down. $T_{\rm max}\sim 1\,\mathrm{TeV}$ does not pose any direct threat to the standard Big Bang, though it severely constrains the building of models of inflation or baryogenesis. Therefore, provided it existed, an alternative description valid in the regime $T\gg 1\,\mathrm{TeV}$ would undoubtedly be worth investigating. The purpose of this paper is to do so in the RS model, for which such a high energy description is offered by the AdS/CFT correspondence [6].

According to the AdS/CFT interpretation [7] the RS model represents a quasi-conformal, strongly coupled 4D-field theory coupled to 4D-gravity. In this view, the KK resonances, the graviton zero mode excluded, and the Standard Model particles are just bound states of this purely four dimensional theory [8]. This viewpoint clarifies the puzzles raised by the RS model. For instance, the CFT picture explains in a simple way why an observer at the SM boundary sees gravity becoming strong at $1\,\mathrm{TeV}$, while an observer at the other boundary only experiences quantum gravity at energies of order $10^{19}\,\mathrm{GeV}$: the observer at the SM boundary is part of the CFT, the strong coupling he experiences at $1\,\mathrm{TeV}$ is analogous to what pions go through at $1\,\mathrm{GeV}$ and is not truly due to gravity becoming strong. The CFT interpretation of the RS model is valid pretty much for the same reasons that gravity on full AdS_{d+1} space describes a d-dimensional CFT without gravity [9]. Several qualitative and quantitative checks of the consistency of the holographic interpretation have been given [10, 11, 8, 12]. In absence of a stabilization mechanism, the RS model is a CFT where dilation invariance is spontaneously broken, the radion μ being the corresponding Goldstone boson (dilaton). The GW stabilization mechanism is holographically dual to turning on a quasi marginal deformation of the CFT which generates a small weak scale by dimensional transmutation. This is analogous to what happens in technicolor models.

In this paper we study the finite temperature behavior of the RS model from the holographic perspective. In the gravity picture now time is euclidean and compactified on a circle, while the 5D-gravitational action is interpreted as the free energy of the corresponding 4D theory. We argue, as suggested in ref. [8], that at high enough temperature the model is in a phase described on the gravity side by AdS_5-Schwarzschild (AdS-S), with the TeV brane replaced by the black-hole horizon. This is in close analogy to what happens in the standard AdS/CFT correspondence at finite T [13]. Indeed in absence of a stabilization of the 5th dimension the dual 4D theory is conformal (though spontaneously broken) and there is no distinction between high and low temperature: at the gravity level it is described by AdS-S at any finite temperature. This picture is based on some test calculation and on entropic considerations. For example we calculate around the RS classical solution with the TeV brane the leading thermal correction to the radion potential in the regime $\mu \gg T$. In the absence of stabilization, we find that μ is pushed towards $\mu \sim T$ where 5D quantum gravity effects becomes strong and where we expect the AdS-S black-hole to be formed. On the other hand when the radion is stabilized, at low enough temperature, the usual RS solution with the TeV brane and no horizon is perturbatively stable. The action of this solution (free energy) essentially equals the GW potential at its minimum. Comparing the action of the two solutions we determine the critical temperature T_c at which a first order phase transition takes place: for $T > T_c$ the theory is in the hot conformal phase (AdS-S) while for $T < T_c$ the theory is in the SM phase (RS model with TeV brane). We find that T_c is somewhat lower that the weak scale unless the GW scalar background causes a big distortion of the RS metric. As we discuss, such a low critical temperature causes the phase transition to the SM phase as the Universe cools down from a primordial hot CFT phase to be very slow. In the simplest implementation of the GW mechanism [5], unless the 5D Planck scale M, the AdS curvature $1/L$ and the GW scalar profile $\phi^{2/3}$ are comparable, thus spoiling the small curvature expansion, the Universe inflates in the CFT forever as in the old inflation scenario. In this case, even if a high temperature description of the RS model is available, to obtain a viable cosmology we are forced to suppose that the Big Bang temperature does not exceed the weak scale, similarly to the models with flat TeV scale extra dimensions. This behaviour can be traced back to the presence, in the minimal GW scenario, of a secondary minimum at $\mu \sim 0$. By modifying the stabilization mechanism, we can obtain radion potentials without a secondary minimum: in this case we have a cosmological scenario in which, after a long period of inflation, the phase transition finally happens. Even if the reheating temperature remains bounded by the TeV scale, we may have a predictive cosmological evolution starting from temperatures as high as the Planck scale.

This paper is organized as follows. In section 2 we give our basic notation and conventions and present an illustrative computation of the radion effective potential at low temperature. In sections 3 to 6 we discuss the two gravitational solutions that are relevant at finite T, including the effects of a GW scalar field. In sections 7 and 8 we study the dynamics of the phase transition. We give a rough estimate of the rate of bubble nucleation and compare it to the expansion rate of the Universe. In section 9 we discuss our results. In the appendix we present instead a solution where a "fake" horizon is hidden behind the TeV brane. This solution, though it corresponds to zero temperature (no horizon), it gives the same FRW evolution of a radiation dominated Universe: this result is easily understood as due to the conformally coupled radion.

2. The RS model at finite temperature

We first briefly recall the definition of the model and establish our notation. The fifth dimension is compactified on an $S_1/\mathbb{Z}_2$ orbifold. We parameterize the single covering of $S_1/\mathbb{Z}_2$ with a coordinate $z \in [z_0, z_1]$. The 4D subspaces at the z_0 and z_1 boundaries are respectively the Planck brane and the TeV brane. The pure gravity part of the action is

$$S = 2M^3 \int d^4x dz \, \left[\sqrt{-g}(R + 12k^2) - 12k\sqrt{-g_0}\, \delta(z - z_0) + 12k\sqrt{-g_1}\, \delta(z - z_1)\right], \quad (2.1)$$

where M is the 5D Planck mass, $k = 1/L$ is the AdS curvature, g_0 and g_1 are the induced metric at the two boundaries and the associated terms can be interpreted as the tension of the corresponding branes. Then the metric

$$ds^2 = e^{-2kz}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2 \quad (2.2)$$

solves Einstein's equations over the full space, including the jump discontinuities at the boundaries[1]. Due to the warp factor in the metric, a 4D theory localized at z_1 experiences a redshift factor $e^{-k(z_1 - z_0)}$ in its energy scales with respect to a 4D theory at z_0. Conversely, as the 4D graviton mode is localized near the Planck brane, the four dimensional Planck scale is not redshifted

$$M_4^2 = M^3 L(e^{-2kz_0} - e^{-2kz_1})\,. \quad (2.3)$$

So if the Standard Model is localized at z_1 the electroweak hierarchy can be explained for $(z_0 - z_1)/L \sim 30$ keeping $M \sim 1/L$ [4]. The size of the fifth dimension is however a modulus, corresponding to a massless field: the radion μ. To truly explain the hierarchy this degeneracy must be lifted. The simplest solution to this problem is the GW mechanism that we will discuss later on.

The AdS/CFT correspondence offers a very useful 4D interpretation of the RS model. By the AdS/CFT correspondence, gravity (or better string theory) on full AdS_{p+1} is dual to a p-dimensional conformal field theory without gravity. The correspondence is a duality since in the limit $ML \gg 1$ (and $g_{\text{string}} \ll 1$) where classical gravity is a good description on the AdS side, the corresponding CFT is strongly coupled (large number of states N^2 for large and fixed 't Hooft coupling). One of the essential aspects of this duality is that the isometries of AdS_{p+1} form the group $\mathrm{SO}(p-1, 2)$, which coincides with the conformal group in p-dimensional spacetime. For example the isometry $z \to z + L\ln\lambda$, $x \to \lambda x$ corresponds to a dilation in the p dimensional CFT. In particular notice that a shift to a larger z corresponds to going to larger length scales in the p-dimensional theory. Therefore from the holographic viewpoint going to smaller (larger) z is equivalent to RG evolving towards the UV (IR) in the lower dimensional theory. In this respect the RS model looks like a 4D theory well described by a CFT at intermediate energies, but whose UV and IR behavior deviates from a pure CFT because of the presence of respectively the Planck and TeV brane. Now, by taking the limit $z_0 \to -\infty$, the Planck brane is (re)moved to the boundary of AdS. In this limit $M_4 \to \infty$ and 4D gravity decouples. However the spectrum of KK gravitons is not much affected, their mass splittings being set by the radion vacuum expectation value (VEV) $\langle\mu\rangle = ke^{-kz_1}$. Unlike in the ADD scenario, there remains no 4D long distance force mediated by spin 2 particles, as if there was really no gravity. This is consistent with a purely 4D interpretation of the model. The resulting model with just the TeV brane is directly interpreted as a 4D theory where conformal invariance is

[1]For more general solutions see appendix.

spontaneously broken by the VEV of the radion. Notice for instance that $\langle\mu\rangle = ke^{-kz_1}$, which fixes the mass scale, transforms with conformal weight 1 under the AdS dilation isometry mentioned above: a clear sign of spontaneous breaking. Finally, bringing in from infinity the Planck brane with its localized graviton zero mode corresponds to gauging 4D gravity in this CFT.

Let us now study the thermal properties of the model. While at zero temperature $\mu = ke^{-kz_1}$ is a flat direction, at $T \neq 0$ we expect a potential to be generated. For simplicity let us decouple 4D gravity by moving the Planck brane towards $z = -\infty$ and consider a toy model where the TeV brane is empty. To further simplify let us also focus on $T \ll \mu$. In this limit the KK are too heavy to be thermally excited and the radion is the only relevant degree of freedom. Virtual KK exchange is however the source of radion interaction. In order to calculate the radion effective potential we must first integrate out the KK modes to get an effective lagrangian. The leading effects arise from tree level KK exchange. To study fluctuations around the RS metric it is convenient to choose coordinates where the TeV brane is not bent and located at z_1 and the metric given by

$$ds^2 = e^{-2kz-2f(x)e^{2kz}}(\eta_{\mu\nu} + h_{\mu\nu}(x,z))dx^\mu dx^\nu + (1+2f(x)e^{2kz})^2 dz^2 \,. \qquad (2.4)$$

In this particular parameterization there is no kinetic mixing between the scalar perturbation $f(x)$ (radion) and the spin 2 fluctuations $h_{\mu\nu}$ [14]. The kinetic term for the radion turns out to be [15, 16]

$$\mathcal{L}_{\rm kin} = -12(ML)^3 e^{2kz_1} k^2 (\partial_\mu f)^2 \,. \qquad (2.5)$$

Notice that in the standard parameterization the radion μ is given by the warp factor at the TeV brane: $\mu = k\,\exp(-kz_1 - fe^{2kz_1})$. In what follows we will treat f as the quantum fluctuation over the background radion VEV $\langle\mu\rangle = ke^{-kz_1}$. At low energy (low temperature) the leading radion interaction term involves 4 derivatives, and it arises by KK mode exchange in the static limit (neglecting four momentum in their propagator). By substituting eq. (2.4) into eq. (2.1), the relevant action terms are

$$\mathcal{L}(x) = 2M^3 \int dz \left\{ e^{2kz} h_{\mu\nu} T^{\mu\nu} + e^{-4kz}\frac{1}{4}(-\partial_z h^{\mu\nu}\partial_z h_{\mu\nu} + \partial_z h^\mu_\mu \partial_z h^\nu_\nu) \right\} \qquad (2.6)$$

$$T^{\mu\nu}(x) \equiv 2\partial^\mu f(x)\partial^\nu f(x) - 4f(x)\partial^\mu\partial^\nu f(x) + \eta^{\mu\nu}\left(\partial^\rho f(x)\partial_\rho f(x) + 4f(x)\partial^\rho\partial_\rho f(x)\right) \qquad (2.7)$$

The first term in $\mathcal{L}(x)$ describes the interaction between two radions and a graviton, while the second one is the five dimensional contribution to the graviton kinetic term. As expected, $T^{\mu\nu}$ is proportional to the radion stress-energy tensor. The coupling of f to 4D gravity is non minimal because of the term $-\sqrt{-g}\xi R f^2/2$ with $\xi = 1/3$.[2]

By integrating out the gravitons at tree level we get

$$\mathcal{L}_{\rm eff}(x) = 2M^3 \int dz dz' e^{2kz} e^{2kz'} G(z,z') \left[-T^{\mu\nu}T_{\mu\nu} + \frac{1}{3} T^\mu_\mu T^\nu_\nu \right] , \qquad (2.8)$$

[2]Notice that for the field $\mu = k\,\exp(-kz_1 - fe^{2kz_1})$ the non-minimality parameter is [16] $\xi = 1/6$, as required by conformal invariance [12] (see also appendix. The point is that μ transforms with weight 1 under Weyl rescaling, while f transforms non linearly.

where $G(z,z')$ is the scalar Green function over the RS metric satisfying $(\partial_z e^{-4kz}\partial_z)\times G(z,z') = \delta(z-z')$ with Neumann boundary conditions at $z = z_1$. G is given by[3]

$$\begin{aligned} G_<(z,z') &= -\frac{1}{8k}e^{4kz} \qquad z > z' \\ G_>(z,z') &= -\frac{1}{8k}e^{4kz'} \qquad z < z'\,. \end{aligned} \tag{2.9}$$

Finally, integrating over z and z' we arrive at

$$\mathcal{L}_{\text{eff}}(x) = (ML)^3\frac{e^{8kr}}{24}\left[T^{\mu\nu}T_{\mu\nu} - \frac{1}{3}T^\mu_\mu T^\nu_\nu\right], \tag{2.10}$$

which describes a 4-derivative interaction among four radions. Trilinear couplings are absent in our parameterization as there is no radion-graviton kinetic mixing. The leading correction to the radion potential is the thermal average of $-\mathcal{L}_{\text{eff}}$. The only relevant term will be proportional to $\langle f\partial^\mu\partial^\nu f\rangle\langle f\partial_\mu\partial_\nu f\rangle$: all the other averages are not T dependent being proportional to the equations of motion. In terms of the canonically normalized radion $\tilde f$ we have

$$V_T(\mu) = -\frac{1}{(ML)^3}\frac{1}{288}\frac{1}{\mu^4}\langle\tilde f\partial^\mu\partial^\nu\tilde f\rangle\langle\tilde f\partial_\mu\partial_\nu\tilde f\rangle\,. \tag{2.11}$$

Notice that $\langle\tilde f\partial^\mu\partial^\nu\tilde f\rangle$ is, up to T independent pieces, the stress energy tensor of a free scalar. In the end the result is

$$V_T(\mu) = -\frac{1}{(ML)^3}\frac{\pi^4}{194400}\frac{T^8}{\mu^4}\,. \tag{2.12}$$

Eq. (2.12) tells us that for $\mu \gg T$ the TeV brane is destabilized and pushed by thermal effects towards the AdS horizon.[4] This result could have been anticipated. The T^8/μ^4 behavior is both due to conformal invariance (or, equivalently, dimensional analysis) and to the 4-derivative character of the interaction. The minus sign is due to the attractive nature of forces mediated by spin 2 particles. As μ is pushed below T, KK-gravitons start being thermally produced. In the regime $T > \mu > T/(ML)$ the free energy goes roughly like $-(T/\mu)T^4 \sim T^5$, where T/μ is the number of thermally excited gravitons. When $ML\mu = \tilde M_5 \sim T$, perturbation theory breaks down: for such value of μ the temperature is planckian for a TeV brane observer. Equivalently, the length e^{-kz_1}/T of the time cycle in euclidean space becomes planckian. To summarize: the radion thermal potential has the form $V = T^4 g(T/\mu)$, it is monotonically decreasing with μ in the regime $\mu > T/(ML)$, and therefore thermal effects drive the RS model into the planckian regime. We conclude that around the RS solution thermal equilibrium is either impossible or described by messy planckian physics.

Fortunately gravity itself provides an elegant way out of this situation. As pointed out by Hawking and Page [18], the canonical ensemble of AdS space is described by the

[3]The normalization is the one for the double covering of the $(-\infty, z_1)$ region.

[4]This instability is somewhat related to the Jeans instability of flat space at finite temperature. Also the quantum instability we will describe in the following may be compared to the quantum production of black holes from flat space. These gravitational instabilities at finite temperature are analyzed in [17].

AdS-Schwarzschild (AdS-S) solution. In this case Hawking radiation from the black-hole horizon allows a hot bulk to be in equilibrium. It is then natural to assume that AdS-S will also describe the thermal phase of the RS model with the TeV brane replaced by the horizon. The qualitative picture is then the following: thermal radiation falls with the TeV brane towards $z = \infty$ until the energy density (see discussion above) is so large that everything collapses to form a black hole [19]. This way we do not have to worry about planckian physics as the Schwarzschild horizon acts as a censor.

We now discuss this in more detail also considering the relevant case of a stabilized radius.

3. Two gravity solutions

Let us restart and consider the RS model at finite temperature from the viewpoint of the AdS/CFT correspondence. AdS/CFT relates a conformal theory defined on a d-dimensional manifold $\mathcal{M}$ to gravity (string theory) defined on the product of a $(d+1)$-dimensional space $\mathcal{X}$ and a compact manifold $\mathcal{W}$. $\mathcal{X}$ is an Einstein manifold with negative cosmological constant and its boundary must coincide with $\mathcal{M}$. The precise relationship between the two theories is given through the field-operator correspondence [9]

$$\left\langle \exp \int_{\mathcal{M}} \phi_0 \mathcal{O} \right\rangle_{\mathrm{CFT}} = Z_S(\phi_0) \,, \tag{3.1}$$

where the field ϕ on the gravity side corresponds to the operator $\mathcal{O}$ in the CFT and it reduces to ϕ_0 on the boundary of $\mathcal{X}$. The string partition function Z_S is replaced at low energies by the corresponding (super)gravity quantity. The crucial remark now [9, 13] is that when several spaces $\mathcal{X}_i$ have the same boundary $\mathcal{M}$ the partition function Z_S must be replaced by a sum over all these possibilities. This point is used in [9, 13] to give the dual gravitational description of the deconfining phase transition of a CFT defined on a sphere; two different gravitational descriptions are relevant: one of them gives the dominant contribution to the partition function at high temperature, while the other dominates at low temperature. The transition between these two gravitational regimes was described by Hawking and Page [18] in the context of quantum gravity and is now seen as dual to the CFT phase transition.

Similar considerations turn out to be relevant in our problem. At finite temperature the manifold $\mathcal{M}$ is in our case $\mathbb{R}^3 \times S^1$ and two different "bulk" spaces must be considered: one is the RS solution with TeV brane and time compactified (we keep the Planck brane at infinity), while the other is the AdS-Schwarzschild solution (AdS-S). These are the only solutions describing states of thermodynamical equilibrium: there are more generic non-thermal solutions as we will see in the appendix. The AdS-S euclidean metric is

$$ds^2 = \left(\frac{\rho^2}{L^2} - \frac{\rho_h^4/L^2}{\rho^2}\right) dt^2 + \frac{d\rho^2}{\frac{\rho^2}{L^2} - \frac{\rho_h^4/L^2}{\rho^2}} + \frac{\rho^2}{L^2} \sum_i dx_i^2 \tag{3.2}$$

for $\rho_h \leq \rho < \infty$. For $\rho_h = 0$ this reduces to the pure AdS metric

$$ds^2 = \frac{\rho^2}{L^2}\left(dt^2 + \sum_i dx_i^2\right) + \frac{L^2}{\rho^2}d\rho^2\,, \tag{3.3}$$

equivalent to (2.2) with $\rho = L\exp(-z/L)$.

This solution describes a black hole in AdS space with event horizon situated at[5] $\rho = \rho_h$. The metric (3.2) is solution of the Einstein equations only for a specific value of the time periodicity β,

$$\beta^{-1} = \frac{\rho_h}{\pi L^2} \equiv T_h\,; \tag{3.4}$$

the temperature $T = \beta^{-1}$ must be equal to the Hawking temperature of the black hole. For $T \neq T_h$ a conical singularity arises at the horizon.

The real time 4D interpretation of this solution is quite simple [10]: the Universe is filled with a thermal CFT state with temperature T_h. The energy and entropy of the heat bath are described by the corresponding quantity of the black hole. Moreover if the space is ended at some $\rho_0 < \infty$ by the Planck brane, then ρ_0 has to depend on the proper time on the brane in order to satisfy the Israel junction conditions. This way the induced metric on the Planck brane varies according to a radiation dominated Universe.

A phase transition will occur between the two regimes: there is no real time evolution between the two gravitational solutions as somehow hinted in [8]. Note that our problem is similar to the transition between AdS and AdS-S described by Witten [13]: there the explicit breaking of conformal invariance is given by the radius of the 3-sphere on which the CFT lives, while in our case it is given by the GW mechanism. Being interested in understanding which of the above bulk solutions dominates the partition function at a generic temperature T, in the following sections we compute the free energy of both solutions including the effects of a GW scalar field.

4. The free energy of the black hole solution

We proceed to compute the free energy of the AdS-S solution (3.2) at temperature T. As the following results will be useful when studying the dynamics of the phase transition, we allow the horizon to have a generic Hawking temperature T_h, not necessarily equal to the real temperature T. In euclidean coordinates this leads to a conical singularity localized at the horizon. In fact, after the substitution $(\rho - \rho_h)/\rho_h = y^2/L^2$, by keeping only the leading term in y near the horizon and by neglecting the spatial coordinates in (3.2) one gets

$$ds^2 = \frac{4\rho_h^2 y^2}{L^4}dt^2 + dy^2\,, \tag{4.1}$$

which is the metric of a cone of angle α, with $\sin\alpha = \rho_h/\pi T L^2 = T_h/T$. The integral of the scalar curvature extended to a neighborhood of the horizon can be evaluated by

[5]Notice that putting a 3+1 brane at the Schwarzschild horizon is equivalent to putting none. This is simply because, being infinitely redshifted, it would not affect the dynamics. This is very clear in euclidian space, where the 4-brane would be shrunk to a 3-dimensional plane.

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regularizing the cusp with a spherical cap of small radius r, area $2\pi r^2(1-\sin\alpha)$ and constant curvature $2/r^2$. The integral is thus r-independent and equals $4\pi(1-\sin\alpha)$. From the gravity action (2.1) we see that the contribution of the conical singularity to the free energy density (per 3D volume) is

$$F_{\rm cone} = -T\, 8\pi M^3 \left(1-\frac{T_h}{T}\right)\frac{\rho_h^3}{L^3} = 8\pi^4\,(ML)^3\,T_h^4\,\left(1-\frac{T}{T_h}\right), \tag{4.2}$$

where we used the fact that the free energy is $-T\cdot S$.

The remaining part of the free energy has to be computed as difference of the gravitational action (2.1) between the AdS-S solution and pure AdS [13]. Both actions are divergent, but one can cut the ρ-integral off at $\rho=\Lambda$, take their difference (which is finite) and then let Λ go to infinity. In performing this limit we can neglect the contribution at the boundary (the surface integral of the trace of the extrinsic curvature) because it gives a contribution to the difference $F_{\rm AdS}-F_{\rm AdS-S}$ which goes like ρ_h^8/Λ^4 and vanishes when $\Lambda\to\infty$. The same result can be obtained regularizing the solutions with the explicit introduction of the Planck brane. Einstein equations force the curvature R to equal $-20/L^2$, so the action reduces to

$$S = 2M^3\int d^5x\,\sqrt{-g}(R+12k^2) = -\frac{16M^3}{L^2}\int\sqrt{-g}\,d^5x\,, \tag{4.3}$$

thus giving

$$\begin{aligned} F_{\rm AdS-S} &= \frac{16TM^3}{L^2}\int_0^{1/T'} dt\int_{\rho_h}^{\Lambda} d\rho\frac{\rho^3}{L^3} \\ F_{\rm AdS} &= \frac{16TM^3}{L^2}\int_0^{1/T} dt\int_0^{\Lambda} d\rho\frac{\rho^3}{L^3} \end{aligned} \tag{4.4}$$

as contributions to the free energy densities. Note that the two time integrals above are extended to different domains: in order to compare the two solutions one has to impose that the geometry induced on the cutoff surface $\rho=\Lambda$ is the same [13]. This implies the following relation:

$$\sqrt{\frac{\Lambda^2}{L^2}-\frac{\rho_h^4/L^2}{\Lambda^2}}\,\frac{1}{T'} = \frac{\Lambda}{L}\frac{1}{T} \quad\Rightarrow\quad \left(1-\frac{\rho_h^4}{2\Lambda^4}\right)\frac{1}{T'} \simeq \frac{1}{T}\,. \tag{4.5}$$

The difference between the two free energies is thus given by

$$F_{\rm AdS-S}-F_{\rm AdS} = \frac{16M^3}{L^2}\left[\frac{1}{4L^3}(\Lambda^4-\rho_h^4)-\frac{\Lambda^4}{4L^3}\left(1-\frac{\rho_h^4}{2\Lambda^4}\right)\right] = -2\pi^4(ML)^3T_h^4\,. \tag{4.6}$$

Adding the singular contribution (4.2) we get the total F for the black hole solution,

$$F = 6\pi^4(ML)^3T_h^4 - 8\pi^4(ML)^3TT_h^3\,, \tag{4.7}$$

where we can distinguish the energetic and the entropic contribution. Obviously F is minimum for $T_h=T$: the Einstein equations forbid the conical singularity in the absence of a source term. The value of F at $T_h=T$ is

$$F_{\rm min} = -2\pi^4(ML)^3T^4\,. \tag{4.8}$$

By holography, this equation is interpreted as the free energy of a strongly coupled large N CFT, with $N^2 = 16\pi^2(ML)^3 + 1$ [10].

Eq. (4.8) represents the classical gravity action. However even after including quantum (or string) corrections the action will keep going like T^4. This is because T is not reparametrization invariant. In particular under the rescaling $(x, t, 1/\rho) \to \lambda(x, t, 1/\rho)$ we have $T \to T/\lambda$ so that the action $\propto \int T^4 d^4x$ is invariant, but any other power of T would not. Then graviton loops just correct F by factors of $1/(ML)^3 \sim 1/N^2 \ll 1$.

One can reason similarly for the RS solution with the TeV brane at $\rho = \rho_1 = Le^{-kz_1}$. At the classical level one easily finds $F_{\rm RS} - F_{\rm AdS} = 0$.[6] Then eq. (4.8) gives truly $F_{\rm AdS-S} - F_{\rm RS} \simeq -(\pi^2/8)N^2T^4$ so that at the level of the classical solutions AdS-S is thermodynamically favored at any temperature $T > 0$. As we discussed in the previous section, higher order corrections will give rise to a non trivial $F_{\rm RS}(T, \mu) = T^4 f(T/\mu)$. Two possibilities are then given. In the first, $F_{\rm RS}$ does not even have a stationary point in μ, in which case AdS-S is the only possible phase. In the second, the stationary value $\langle\mu\rangle$ is given by the only reasonable scale in the problem, the scale at which physics becomes planckian and quantum corrections are unsuppressed $\langle\mu\rangle \sim T/(ML)$. In this case the free energy will be very roughly given by the number $T/\mu = ML$ of excited KK modes: $F_{\rm RS} \sim -(ML)T^4 \sim -N^{2/3}T^4$. At large N also this second possibility is thermodynamically disfavored with respect to AdS-S.

We will now see how radius stabilization changes the picture.

5. The effect of the Goldberger-Wise field

The radion modulus of the RS model must be stabilized in order to get a viable solution to the hierarchy problem. Goldberger and Wise [5] have shown that this can be done without fine tunings through the introduction of a bulk scalar field ϕ. Its action is given by

$$S_{\rm GW} = \int d^4x d\rho \Big\{ \sqrt{-g}\left[-g^{MN}\partial_M\phi\partial_N\phi - m^2\phi^2\right] + \delta(\rho-\rho_0)\sqrt{-g_0}\mathcal{L}_0 + \\ + \delta(\rho-\rho_1)\sqrt{-g_1}\mathcal{L}_1 \Big\} . \tag{5.1}$$

The brane lagrangians $\mathcal{L}_{0,1}$ are assumed to force ϕ to have the boundary values $\phi(\rho_{0,1}) = L^{-3/2}v_{0,1}$. The general ρ-dependent solution of the equations of motion is

$$\phi(\rho) = A\rho^{-4-\epsilon} + B\rho^{\epsilon} , \tag{5.2}$$

where $\epsilon = \sqrt{4 + m^2L^2} \simeq m^2L^2/4$ for a small mass. A and B are fixed by the boundary conditions on the branes. The induced 4D potential for the radion field $\mu \equiv \rho_1/L^2$ is [5]

$$V_{\rm GW}(\mu) = \epsilon v_0^2\mu_0^4 + \left[(4+2\epsilon)\mu^4(v_1 - v_0(\mu/\mu_0)^{\epsilon})^2 - \epsilon v_1^2\mu^4\right] + \mathcal{O}(\mu^8/\mu_0^4) , \tag{5.3}$$

[6] This corresponds to the cosmological constant being tuned to zero and the radion potential being flat in the RS model.

where $\mu_0 = \rho_0/L^2$ and we have assumed $|\epsilon| \ll 1$. For $\epsilon > 0$ the potential above has a global minimum at

$$\mu = \mu_{\rm TeV} \equiv \mu_0 \left(\frac{v_1}{v_0}\right)^{1/\epsilon} \left[\frac{8 + 6\epsilon + \epsilon^2 + (2+\epsilon)\sqrt{4\epsilon + \epsilon^2}}{8 + 8\epsilon + 2\epsilon^2}\right]^{1/\epsilon} . \tag{5.4}$$

The huge hierarchy between the weak and Planck scale can be naturally obtained for parameters not far from 1 (e.g. $v_1/v_0 \sim 1/10$ and $\epsilon \sim 1/20$). For $\epsilon < 0$ the only minimum of the potential (5.3) is at $\mu = 0$. However, with a small change of the TeV brane tension δT_1 the potential is modified by a term $\delta T_1 \mu^4$ which can induce a non-trivial minimum at $\mu \sim \mu_0 (v_0/v_1)^{1/|\epsilon|}$: a viable solution can be obtained for $v_0 < v_1$ [8, 12].

The reason why this mechanism solves the hierarchy problem is that the potential for the radion field is of the form $\mu^4 P(\mu^\epsilon)$, where P is a polynomial: the scale non-invariance is only induced by the slow varying μ^ϵ term. The holographic dual of the GW field is an almost marginal operator $\mathcal{O}$ with conformal dimension $4+\epsilon$; from this point of view the hierarchy problem is solved through the slow RG evolution of $\mathcal{O}$ which dynamically generates a small mass scale $\mu_{\rm TeV}$ [12]. This is the same phenomenon of dimensional transmutation at work in technicolor and in the Coleman-Weinberg mechanism. A completely general potential of the form $\mu^4 P(\mu^\epsilon)$ can be obtained by considering a self interacting GW field [12]; for simplicity we stick to (5.3): further terms would not change our conclusions.

The effects of the introduction of the GW field are calculated neglecting the back-reaction of its stress-energy tensor on the AdS metric. This is allowed for sufficiently small values of ϕ [5], namely

$$v_{0,1} \ll (ML)^{3/2} \sim N\,. \tag{5.5}$$

In the 4D description this is equivalent to the requirement that the deformation of the CFT induced by the operator $\mathcal{O}$ can be treated perturbatively at all scales down to $\mu_{\rm TeV}$.

For $\epsilon > 0$ the operator $\mathcal{O}$ is weak in the IR and gets strong in the UV; a more conventional situation arises for $\epsilon < 0$ in which the deforming operator gets strong in the IR and would eventually completely spoil conformal invariance at a scale

$$\Lambda \simeq \mu_0 \left(\frac{4\pi v_0}{N}\right)^{1/|\epsilon|} \tag{5.6}$$

which is much less than $\mu_{\rm TeV}$ in our weak coupling assumption (5.5).

At low enough temperature the free energy in the phase with the TeV brane is well approximated by (5.3): it will now surely have a stationary point (a minimum indeed). A phase transition will then happen if the RS-GW free energy becomes lower than that for AdS-S.

Note that eq. (5.3) has a secondary local minimum at $\mu = 0$, which describes a full AdS space. Full AdS is stable at $T = 0$ also with respect to formation of the black hole as evident form eq. (4.7). Indeed we may consider the case of $\mu = 0$ to coincide also with the $T \to 0$ limit of AdS-S. In the case $\epsilon < 0$ we have seen that we are forced to add a contribution $\delta T_1 \mu^4$ to have a minimum in the TeV region: this term must satisfy $\delta T_1 < \epsilon v_1^2$ to get the minimum and $\delta T_1 > -(4+\epsilon) v_1^2$ to have a potential bounded from below. Even if we cannot make explicit calculations, we expect that a secondary local minimum is always present in the small μ region: the free energy will be modified under the scale Λ by contributions of the order Λ^4, so that the maximum of the GW potential at $\mu \lesssim \mu_{\rm TeV}$ will always be high enough to give a stable minimum near the origin. For $\epsilon > 0$ the presence of a second minimum at $\mu \simeq 0$ is not compulsory: adding the term $\delta T_1 \mu^4$ to eq. (5.3) with $\delta T_1 < -(4+\epsilon)v_1^2$ the GW potential remains bounded from below but has a unique minimum.

In order to compare the two free energies we must first take the effect of the GW field in the AdS-S background into account . This corresponds to evaluating the GW scalar action at the stationary point around the AdS-S background. Notice that the action now contains only the contribution of the Planck brane:

$$S_{\rm GW} = \int d^4x d\rho \left\{ \sqrt{-g}\left[-g^{MN}\partial_M\phi\partial_N\phi - m^2\phi^2\right] + \delta(\rho-\rho_0)\sqrt{-g_0}\mathcal{L}_0 \right\}. \tag{5.7}$$

The boundary condition at the TeV brane is replaced with the requirement that the solution is regular at the Schwarzschild horizon. Setting $L = 1$ the equation of motion for a ρ-dependent ϕ in the metric (3.2) is

$$(\rho^5 - \rho\rho_h^4)\partial_\rho\partial_\rho\phi + (5\rho^4 - \rho_h^4)\partial_\rho\phi - m^2\rho^3\phi = 0\,. \tag{5.8}$$

With the change of variable $z \equiv (\rho/\rho_h)^4$ we obtain the hypergeometric equation

$$z(1-z)\phi'' + (1-2z)\phi' + \frac{m^2}{16}\phi = 0\,. \tag{5.9}$$

The equation is invariant under $z \to 1-z$ and its solutions are [20]

$$F(\alpha,\beta,1,z) \qquad \text{and} \qquad F(\alpha,\beta,1,1-z)\,, \tag{5.10}$$

with α and β solutions of the quadratic equation $x^2 - x - m^2/16 = 0$. Hypergeometric functions have a cut on the real axis from 1 to $+\infty$, so that only the second solution is regular at the horizon; the first one diverges logarithmically and gives a divergent contribution to the action. The regular solution behaves asymptotically ($\rho \gg \rho_h$) as

$$\phi(\rho) \simeq A\left[(1+\mathcal{O}(\epsilon^2))(\rho/\rho_h)^{\epsilon} - \frac{\epsilon}{8}(\rho/\rho_h)^{-4+\epsilon} + \frac{\epsilon}{8}(\rho/\rho_h)^{-4-\epsilon}\right]. \tag{5.11}$$

Note the presence of the second term, which is absent in the solution over pure AdS of eq. (5.2). The overall factor A is fixed by requiring $\phi(\rho_0) = v_0/L^{3/2}$, as forced by $\mathcal{L}_0$. Upon integration by parts, the GW action (5.7) is just due to boundary terms, as the bulk term vanishes on the equations of motion. Moreover, the contribution at the horizon vanishes for the regular solution since $g^{\rho\rho}|_{\rho_h} = 0$. Then, after properly rescaling the temperature according to eq. (4.5), we find that the GW field induces a correction

$$\Delta F_{\rm GW} = \sqrt{-g}\; g^{\rho\rho}\partial_\rho\phi\;\phi\Big|_{\rho_0} = \epsilon v_0^2\mu_0^4 - \epsilon\frac{\pi^{4+2\epsilon}}{2}v_0^2 T_h^4\left(\frac{T_h}{\mu_0}\right)^{2\epsilon} \tag{5.12}$$

to the AdS-S action in eq. (4.8). The first term can be, as always, canceled by a local counterterm which redefines the Planck brane tension; the other term depends as expected only on the RG-invariant quantity $v_0\mu_0^{-\epsilon}$.

6. Computation of the transition temperature

We have now all the ingredients to compare the free energies for the two solutions and find in which regime each one is dominant. We refer for simplicity to the original GW case $\epsilon > 0$ and with no further modifications of the TeV brane tension; similar results hold in other cases ($\epsilon < 0$). The contribution $\epsilon v_0^2 \mu_0^4$ due to the boundary term of the GW field on the Planck brane is common to the two solution and cancels out. When the temperature is sufficiently below the weak scale, the free energy in the RS case is well approximated by the GW potential (5.3), so that we have to compare

$$F_{\mathrm{RS}} = \left[(4+2\epsilon)\mu^4 (v_1 - v_0(\mu/\mu_0)^\epsilon)^2 - \epsilon v_1^2 \mu^4\right] + \mathcal{O}(T^4) \tag{6.1}$$

$$F_{\mathrm{AdS-S}} = 6\pi^4 (ML)^3 T_h^4 - 8\pi^4 (ML)^3 T T_h^3 - \epsilon \frac{\pi^{4+2\epsilon}}{2} v_0^2 T_h^4 \left(\frac{T_h}{\mu_0}\right)^{2\epsilon} . \tag{6.2}$$

The last term in the AdS-S case can be neglected in the weak GW coupling limit of eq. (5.5).

The value of the GW potential at the minimum (5.4) is

$$V_{\min} \simeq -\epsilon\sqrt{\epsilon} v_1^2 \mu_{\mathrm{TeV}}^4 \tag{6.3}$$

which has to be compared with the minimum of $F_{\mathrm{AdS-S}}$ (4.8). These quantities will be equal at a critical temperature T_c given by

$$-\epsilon^{3/2} v_1^2 \mu_{\mathrm{TeV}}^4 = -2\pi^4 (ML)^3 T_c^4 \quad \Rightarrow \quad T_c = \left(\frac{8\epsilon^{3/2} v_1^2}{\pi^2}\right)^{1/4} \frac{1}{\sqrt{N}} \mu_{\mathrm{TeV}} . \tag{6.4}$$

Since $\epsilon \sim 1/20$ and since $v_1 \ll N$, in order to have negligible back-reaction (see (5.5)), we have

$$T_c \ll \mu_{\mathrm{TeV}} . \tag{6.5}$$

At $T < T_c$ the system is in the RS phase. At $T = T_c$ the system undergoes a first order phase transition to a hot conformal phase. Notice that by the low value we found for T_c our neglect of thermal corrections to eq. (6.1) was justified. However, when there is a large number g_* of light degrees of freedom localized on the TeV brane, their contribution to the RS free energy $\sim -g_* T^4$ may be relevant and T_c increased. As we will discuss later on g_* in the SM is not large enough to dramatically change the picture. Therefore we will stick to the simple case where the TeV brane is empty.

We conclude that if the Universe was ever heated at temperatures well above the weak scale μ_{TeV}, its expansion would be driven in that regime by hot CFT radiation. The transition to the RS solution would only happen at a temperature below T_c. In the case of a small back reaction, eq. (6.5) implies that the KK gravitons which represent the collective excitations of the broken CFT and whose lowest mass is $\mathcal{O}(\mu_{\mathrm{TeV}})$ are never thermally excited. The Universe never experiences a 5D behavior.

7. Dynamics of the phase transition

The two gravitational solutions we described are local minima of the free energy: we expect a first order phase transition proceeding, from a 4-dimensional perspective, through bubble nucleation. The bubble will interpolate between the unbroken hot CFT at infinity

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and the broken phase inside. From the 5-dimensional perspective we view this process as the formation of spherical brane patches on the horizon. These expand and eventually coalesce to form a complete 3-brane.

For the cases in which there is a secondary local minimum at $\mu = 0$, in order to have a viable cosmology we must require that the rate of bubble nucleation per unit volume Γ is larger than H^4 at $T \sim T_c$. If this were not true, then the Universe would cool down below T_c and begin to inflate. This is because the cosmological constant for the false vacuum is positive if we assume it to be zero in the true RS vacuum. Thus $\Gamma < H^4$ corresponds to an old inflation scenario, which is ruled out because of its inefficient reheating [21, 22]. In this case of small transition rate the Universe would asymptotically approach the local minimum at $\mu = 0$. This asymptotic cold state is not possible for the cases where there is a unique minimum of the potential: a small transition rate will cause a period of inflation, but at sufficiently low temperature the Universe will evolve towards the unique minimum. We will come back to this possibility in the conclusions. In what follows we study the transition rate and the cosmology for the case in which there is a secondary local minimum.

In order to estimate Γ it is reasonable to neglect 4D gravity by moving the Planck brane to infinity. In the semiclassical approximation to evaluate Γ one should normally find the 5D gravity solution corresponding to the bounce. We will not undertake this task and not just because it is technically hard. The main reason is that the exact bounce solution would necessarily lead us out of the limited domain where the effective field theory description of the RS model applies. As it will be clarified below, by topology the bounce will involve regions of space where the euclidean time cycle is of (sub)planckian length: here quantum gravity effects are unsuppressed and we also expect the unknown physics that resolves the brane to matter. However we will argue that when the critical temperature is low enough, the dominant contribution to the tunneling amplitude will come from the region $\mu \gg T$ where effective field theory applies and where the radion is the only relevant mode. In this regime we will be able to give a reasonable estimate of Γ.

Let us now consider the general features of the bounce. Note, first of all, that the two gravitational solutions we want to interpolate between have different topologies. The AdS-Schwarzschild is equivalent to the product of a disk (ρ is the radial variable and t is the angular one) bordered by the Planck brane and $\mathbb{R}^3$ (the three spatial dimensions); to obtain the RS space we have to insert the TeV brane: we cut a hole in the ρ - t disk making an annulus. So AdS-S is simply connected while RS is not. It is well known that in General Relativity a topology change can be accomplished without generating a configuration of infinite action. In fact the two solutions can be deformed one into each other by moving the horizon towards $\rho = 0$ and then moving the brane back from $\rho = 0$. In this deformation we pass through ordinary AdS space with periodic time. This has the same topology as AdS-S except that all the points at $\rho = 0$ are identified. All the configurations along the path have finite action per unit volume. Therefore we find it plausible to assume that there will exist a bubble solution (instanton) performing such an interpolation: in moving from the outside to the center of the bubble we see the horizon going towards $\rho = 0$, we arrive at the pure AdS and then the brane comes in from $\rho = 0$ to a finite value, as shown in figure 1. In figure 2 we show what the bubble looks like topologically.

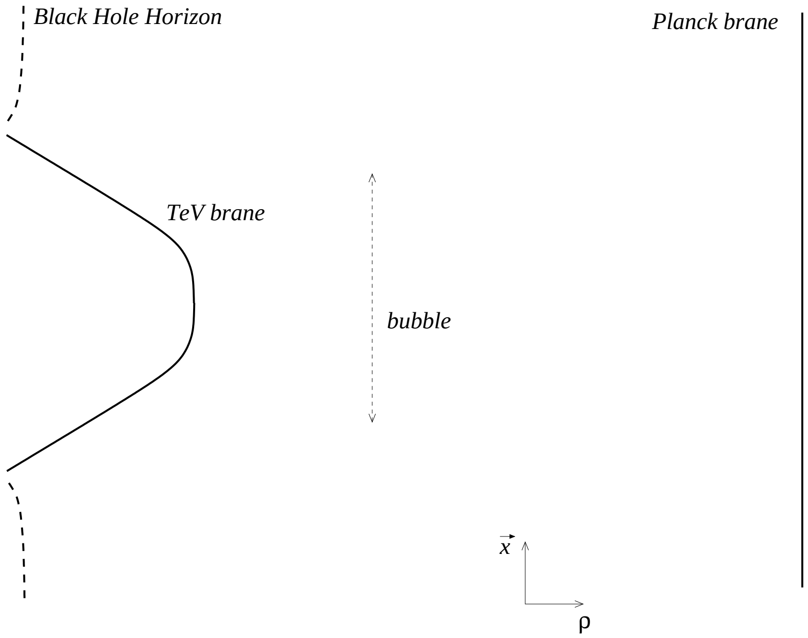

Figure 1: 5D picture of the 4D bubble configuration. The two solutions are connected by moving the black hole horizon and the TeV brane towards the AdS infinity.

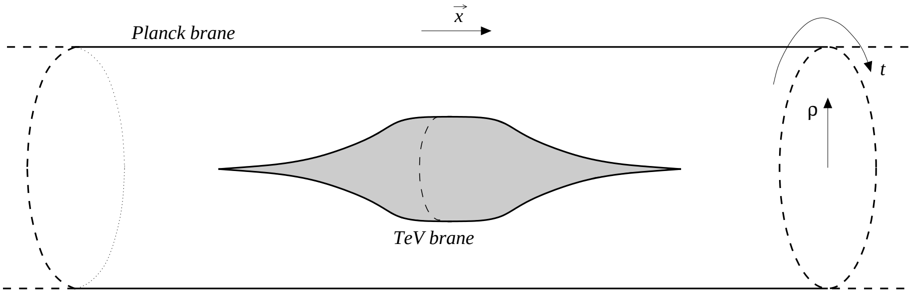

Figure 2: The topology of the bubble. We have suppressed two spatial dimensions, so that the AdS-S solution is a cylinder: the bubble appears as hole in the cylinder. The rotational invariance is here seen as a discrete $\mathbb{Z}_2$ inversion symmetry.

In other words, we have chosen a particular one-parameter set of configurations: the RS model with generic TeV brane position μ is glued to the set of black hole configurations, parametrized by their Hawking temperature T_h, through the pure AdS solution, which is obtained both for $\mu \to 0$ and for $T_h \to 0$.

A potential for this set of configurations is obtained by joining (6.1) and (6.2) (see figure 3)

$$\begin{aligned} F_{\rm RS} &= \left[(4+2\epsilon)\mu^4(v_1 - v_0(\mu/\mu_0)^\epsilon)^2 - \epsilon v_1^2\mu^4\right] \quad \text{and} \\ F_{\rm AdS-S} &= 6\pi^4(ML)^3T_h^4 - 8\pi^4(ML)^3TT_h^3 \end{aligned} \tag{7.1}$$

(we neglect the GW contribution in the black hole case, as it is subleading in N). This potential should be taken with some caution. Indeed in the region $\mu < T/ML$ quantum gravity corrections are unsuppressed and also our effective field theory description of the

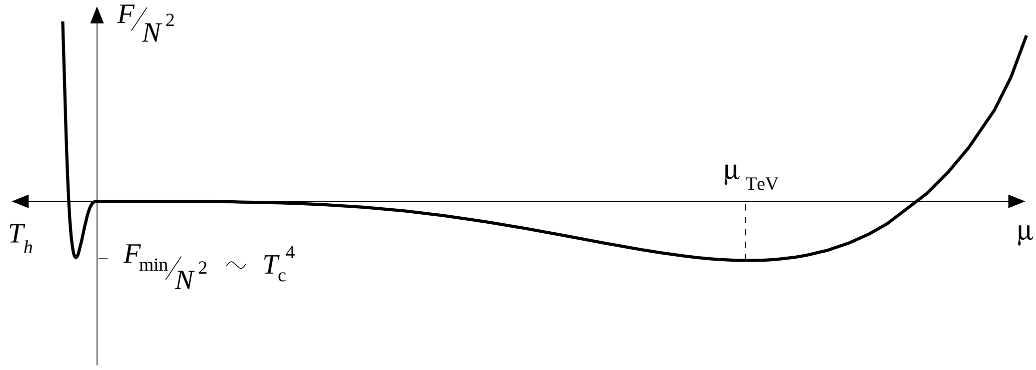

Figure 3: The free energy for the RS solution (right side) and the black hole one (left). RS model is parametrized by the radion VEV μ, while the AdS-S one by the Hawking temperature T_h. The two classes of solutions coincide at $\mu = T_h = 0$.

brane breaks down. In this region we expect the physics that resolves the brane to become important. On the black hole side it is also not obvious that the relevant degree of freedom is simply represented by the horizon position. On the other hand in the region $\mu > T/ML$ the radion (and its potential) gives a good description of the smooth brane deformations, for which $d\mu/dx \ll \mu^2$. When we can treat the brane by effective field theory the radion is also the lightest mode: unlike the KK masses, the GW potential is suppressed by $v_1^2/N^2 \ll 1$. Now, our crucial remark is that in the tractable case where $T_c \ll \mu_{\rm TeV}$, the parametrically dominant contribution to the bubble action comes from the region of large μ, with small $d(1/\mu)/dx$, where we can control our effective action. The modeling of the potential in the region of small μ and on the horizon side will not sizably affect the result. The basic reason for this is gotten by looking at figure 3. The normalized potential is very flat on the brane side, where the depth is only of order $-(v_1/N)^2\mu_{\rm TeV}^4$ over a "large" distance $\sim \mu_{\rm TeV}$. On the black hole side there is no small parameter suppressing the depth. Then in order to balance the gradient energy due to the large distance in μ the bubble will have to be large and therefore most of its action will come from the large μ region. This will become more clear in the quantitative discussion below. Also figure 1 is useful to get an idea: as $T_c \ll \mu_{\rm TeV}$, the brane patch should protrude well out into the bulk, thus dominating the action.

A further point has to be stressed in the case $\epsilon < 0$. In this case the operator which induces the breaking of the conformal symmetry gets strong in the IR and at the scale Λ defined in (5.6) the conformal picture is completely spoiled: correspondingly in the gravity picture the GW field leads to a strong deformation of the pure AdS metric. This implies that in both phases we expect to enter an unknown regime at $T_h < \Lambda$ and $\mu < \Lambda$, where the potential will be of size Λ^4 instead of naively going through zero. However, when $\Lambda \ll T_c$, for which we can perturbatively study the phase transition, the region of field space affected by large non perturbative corrections is small and makes only a small correction to the bounce action, which as we said is dominated by the large μ region. In conclusion we expect that, for a sufficiently weak deforming operator $\mathcal{O}$, the transition has the same characteristics as for $\epsilon > 0$.

We now move to estimate the transition rate. The bubble nucleation rate per unit volume in a first order phase transition can be written

$$\Gamma = \Gamma_0 e^{-S}\,; \tag{7.2}$$

Γ_0 is a determinant prefactor which in our case we expect to be of the order TeV^4. The dominant dependence on the model parameters is encoded in the exponential of the euclidean action S, computed on the bubble solution interpolating between the false and the true minimum. If the temperature is high enough the characteristic length scale of the bubble is much larger than the time radius $1/T$, so that the favored instanton will have an O(3) symmetry and S reduces to S_3/T, where S_3 is the spatial euclidean action. At low temperature an O(4) symmetric instanton will instead be dominant.

A subtle point should be stressed: in order to compute the exact bounce solution one should be able to evaluate the action for a configuration with a space dependent $T_h(\vec{x})$. To do that one should find a complete solution of the Einstein equations describing the bubble. As this is an almost impossible task we must face the fact that we do not know the "kinetic" term of the T_h field. Note that this implies that we are not able to "canonically" normalize the horizontal scale in the left side of figure 3. For the other side of the figure the situation is more conventional, as we know that the radion kinetic term is [15, 16, 12]

$$\mathcal{L}_{\rm kin} = -12\sqrt{-g}(ML)^3(\partial\mu)^2\,. \tag{7.3}$$

The hierarchy $\mu_{\rm TeV} \gg T_c$ between the two characteristic scales in the potential allows to establish the main features of the bounce action S.

Even if we do not know the explicit form of the contribution to the action S from the "black hole" side of the instanton, we know that this is purely gravitational, so that it will be proportional to $(ML)^3 \sim N^2$ and it will be characterized by the unique energy scale $T \sim T_c$. The kinetic term of the radion μ in (7.3) is similarly proportional to $(ML)^3 \sim N^2$, so we can factor N^2 out of the entire action by rescaling the stabilizing GW potential (7.1): $F_{\rm RS} \equiv N^2\tilde{F}_{\rm RS}$, $F_{\rm AdS-S} \equiv N^2\tilde{F}_{\rm AdS-S}$. The rescaled potential $\tilde{F}_{\rm RS}$ has a characteristic horizontal scale $\sim \mu_{\rm TeV} \gg T_c$ and depth $\sim T_c^4$, while, as we said, $\tilde{F}_{\rm AdS-S}$ has T_c as the only typical scale. At leading order in N we will estimate the action S neglecting the width of $\tilde{F}_{\rm AdS-S}$ with respect to the much greater $\tilde{F}_{\rm RS}$ one: the different horizontal scales characterizing the two parts of the potential allows us to disregard, at leading order in N, the unknown "kinetic" term for the black hole horizon.

Within the above approximation, it is useful to rescale the radion field

$$\tilde{\mu} \equiv \mu\,\epsilon^{3/8}\sqrt{\frac{v_1}{N}}\,. \tag{7.4}$$

In terms of $\tilde{\mu}$, the width of the potential $\tilde{\mu}_{\rm TeV} = \mu_{\rm TeV}\epsilon^{3/8}\sqrt{v_1/N}$ also controls its depth $\tilde{F}_{\rm RS\,min} \simeq -\tilde{\mu}^4_{\rm TeV}$. In this way the position of the minimum $\mu = \mu_{\rm TeV}$ is rescaled to the same energy scale of the potential depth $\tilde{F}_{\rm RS\,min} \simeq -\epsilon\sqrt{\epsilon}v_1^2\mu^4_{\rm TeV}/N^2$. To canonically normalize the kinetic term for $\tilde{\mu}$ (see (7.3)) we have to redefine the coordinates: $\tilde{x} \equiv x \cdot 2\pi/\sqrt{3}\cdot\epsilon^{3/8}\sqrt{v_1/N}$. In such a way we have factored out all the relevant parameters from the action S. In the limit of small and high temperature we expect

$$S_4 \sim \frac{N^4 3^2}{(2\pi)^4 \epsilon^{3/2} v_1^2} \tilde{S}_4 \left(\frac{T}{T_c}\right) \qquad\qquad \frac{S_3}{T} \sim \frac{N^{7/2} 3^{3/2}}{(2\pi)^3 \epsilon^{9/8} v_1^{3/2}} \tilde{S}_3 \left(\frac{T}{T_c}\right), \tag{7.5}$$

where the functions $\tilde{S}_{3,4}(T/T_c)$ have no small parameters. The characteristic length scale of the bubble in the original coordinates x is $R_{\text{bubble}} \sim \mu_{\text{TeV}}^{-1} N/(v_1 \epsilon^{3/4})$: the variation of the brane position is slow and the condition $d\mu/dx \ll \mu^2$ is satisfied. For $T \gg R_{\text{bubble}}^{-1} \sim T_c \sqrt{v_1} \epsilon^{3/8}/\sqrt{N}$ we expect the thermal bounce to be favored with respect to the O(4) symmetric solution.

The estimates (7.5) can also be seen in a rather different way. Essentially they are obtained by neglecting all what happens in the unknown region $\mu < T/ML$, as if the black hole minimum were located just at the end of the region under the control of effective field theory. The important point is that the addition of the unknown region of potential can only make the transition slower. This conclusion is obvious if the problem of bubble nucleation is formulated in terms of tunneling through a barrier of potential and gradient energy [23]: our approximation corresponds to setting the gradient energy to zero and the potential to a constant $V_0 \equiv F_{\text{min}}$ in the unknown region. Therefore our estimates give both the transition rate at leading order in N and, disregarding the unknown region of potential, an upper limit on the transition rate itself. This makes more robust the constraints we will derive in the next section requiring the transition to be fast enough to avoid old inflation.

The above results, though qualitative, are quite reasonable. First of all we see that the phase transition gets more and more difficult the larger N: this could be expected as at large N the transition implies a dramatic decrease in the number of degrees of freedom. Also the v_1-dependence is not unexpected: v_1 describes the strength of the stabilization mechanism, or in the 4D language the influence of the deformation operator introduced in the CFT. As only in presence of such a mechanism the transition is possible, we expect that a large v_1 makes the transition faster and faster.

In the following section we show that for the rate of bubble nucleation to be cosmologically acceptable a small N and a strong stabilization mechanism are required.

8. Thin wall approximation

Let us focus first on a temperature T very close to T_c so that the two minima are almost degenerate and the bubble is described by the thin wall approximation [23]. In this limit we can easily estimate the action for a thermal bounce:

$$S_3 = \frac{2\pi}{3} \frac{\left[\int_0^{\mu_{\text{TeV}}} d\mu \sqrt{2F_{\text{RS}}(\mu)}\right]^3}{(\Delta F)^2} \times \left(\frac{3N^2}{4\pi^2}\right)^{3/2}, \tag{8.1}$$

where F_{RS} is shifted to be zero at the true minimum. In the approximation described in the previous section we have neglected the horizontal width of the black hole side potential thus extending the barrier penetration integral only over the RS side. ΔF is the free energy difference between the two minima and the last factor takes into account the non canonical normalization of μ (see (7.3)).

We can estimate the various quantities appearing in (8.1)

$$
\begin{aligned}
\int_0^{\mu_{\rm TeV}} d\mu \sqrt{2F_{\rm RS}(\mu)} &\simeq \sqrt{2}\epsilon^{3/4} v_1 \mu_{\rm TeV}^3 \\
\Delta F &= \epsilon^{3/2} v_1^2 \mu_{\rm TeV}^4 \left[1 - \left(\frac{T}{T_c}\right)^4\right] \\
\frac{S_3}{T} &\simeq \frac{2\pi}{3} 2^{3/2} \left(\frac{\pi^2}{8}\right)^{1/4} \frac{N^{7/2} 3^{3/2}}{(2\pi)^3 \epsilon^{9/8} v_1^{3/2}} \frac{(T_c/T)}{\left[1 - (T/T_c)^4\right]^2} \,.
\end{aligned}
\tag{8.2}
$$

Evaluating the numerical factors we get

$$
\frac{S_3}{T} \simeq 0.13 \times \frac{N^{7/2}}{\epsilon^{9/8} v_1^{3/2}} \times \frac{(T_c/T)}{\left[1 - (T/T_c)^4\right]^2} \,, \tag{8.3}
$$

which shows the correct dependence on N, ϵ and v_1 found in the previous section. We have checked by numerical computations that the thin wall approximation gives a reasonable estimate (modulo a factor ~ 2) of the action until the energy difference of the two minima is of the same order of the barrier height. Therefore the thin wall approximation is suitable for a qualitative discussion.

Taking the (8.3) as an estimate of the leading contribution in N and $1/v_1$, we can infer a cosmological constraint on the parameters of the model. To avoid the old inflation scenario the transition must take place at $T \sim T_c$, so that the thermal bounce is favored and the thin wall approximation is reasonable. Requiring $\Gamma > H^4_{T \sim T_c}$ we have

$$
0.13 \frac{N^{7/2}}{\epsilon^{9/8} v_1^{3/2}} \lesssim 137 \quad \Rightarrow \quad \frac{N}{\epsilon^{9/28} v_1^{3/7}} \lesssim 7 \,, \tag{8.4}
$$

where we used the fact that $(T_c/T)/\left[1 - (T/T_c)^4\right]^2 > 1$ for $T < T_c$ ([7]). Taking $\epsilon \simeq 1/20$ we finally get

$$
\frac{N}{v_1^{3/7}} \lesssim 3 \,. \tag{8.5}
$$

In the above computations we used the specific GW potential with $\epsilon > 0$; other possible solutions can be obtained by modifying the TeV brane tension and/or taking $\epsilon < 0$. We expect that also in these cases the constraint (8.5) will not be significantly modified. For instance, allowing for an extra contribution $\Delta V \sim -\delta T_1 \mu^4$ due to the TeV brane tension one has generically a deeper minimum $V_{\rm min} \sim -\epsilon \delta T_1 \mu_{\rm TeV}^4$. Then the bound above becomes

$$
\frac{N}{\delta T_1^{3/14}} \lesssim 3.7 \,. \tag{8.6}
$$

The bound (8.5) shows that a model with a large N, which in the 5D picture allows to neglect quantum gravity effects in the AdS solution, with a perturbative GW stabilization

[7] We have neglected the logarithmic corrections to (8.4) one would obtain considering that the number of relativistic degrees of freedom g_* is of order N^2.

mechanism ($v_1 \ll N$) seems to have an unviable high T cosmology. The introduction of a stronger stabilization mechanism would partially alleviate the problem, but one would loose the nearly-AdS (or nearly conformal) picture.

Before going to the conclusions, an important comment is necessary. In all the calculations we have done so far we have neglected the contribution to F from the light degrees of freedom confined to the brane. In other words we have neglected the Standard Model! The contribution of the brane excitations to $F_{\rm RS}$ (eq. (6.1)) is given by

$$F_{\rm SM} = -\frac{\pi^2}{90} g_{*\,\rm brane} T^4 \,, \tag{8.7}$$

where $g_* = 106.75$ for the Standard Model particles at high temperatures. For the calculation of the transition temperature it is easy to see that eq. (6.4) holds with the substitution

$$N^2 \to N^2 - \frac{4}{45}\, g_{*\,\rm brane} \;: \tag{8.8}$$

there is a smaller difference in the number of degrees of freedom between the two phases and T_c gets larger. The inclusion of these light brane excitations gives a negligible correction to the transition temperature and to the dynamics of the transition if $g_{*\,\rm brane} \ll 45/4\, N^2$, which holds, in the case of the SM particles, with N much larger than 3. We consider this bound to be rather weak, as for smaller N the gravitational description cannot be trusted. It is interesting to note that an independent limit to the number of light degrees of freedom confined to the brane could derive from some ideas on the irreversibility of the RG flow, which, roughly speaking, indicates that the number of degrees of freedom must decrease going from the UV to the IR [24]. If $g_{*\,\rm brane}$ is large enough the 4D theory we are holographically describing has a number of IR degrees of freedom larger than in the UV. These bounds could constrain the possibility to give a stringy realization of the RS model with a large number of light particles confined on the TeV brane.

9. Conclusions and outlook

We have used the holographic interpretation of the Randall-Sundrum model to study its finite temperature properties. While we think this problem is interesting *per se*, our main motivation is to understand the early cosmology of the model, when the temperature of the universe was of the order or bigger than the electroweak scale. The picture that emerges is that the model is in different phases at low and high temperature, with a first order phase transition at a temperature T_c. At low temperature we have the RS phase, where physics is described by the SM particles and possibly the lightest KK modes. Here the free energy is dominated by the dynamics that stabilizes the radion. In our paper we focused on a minimal GW mechanism, but notice that in general the SM Higgs has a part in this dynamics [12]. So we may more broadly say that in the low temperature phase radion stabilization and electroweak breaking dominate the free energy. In the high temperature phase, the breaking of conformal symmetry by both the presence of the TeV brane (spontaneous) and the presence of the GW coupling (explicit) is screened by thermal effects. The system behaves here like a hot CFT. In the 5d description the TeV brane is replaced with the black hole horizon of AdS-Schwarzschild. Using the gravitational picture we have calculated the critical temperature T_c. We find that when GW mechanism is associate to a weak coupling (small distortion of AdS metric) T_c is parametrically suppressed with respect to the scale $\mu_{\rm TeV}$ that controls the KK splittings. So T_c is not bigger than the Fermi scale, and more likely even somewhat smaller. The phase transition to the low energy RS regime and electroweak breaking happen essentially at once: at $T < T_c$ the gravity solution with stabilized radion and non zero Higgs VEV becomes thermodynamically favored over AdS-S.

We have then studied how the Universe transits to the SM phase as it cools down during expansion. The phase transition is first order, so it proceeds through the nucleation of bubbles, their expansion and final collision. In order to see if this process is fast enough we have estimated the rate of bubble nucleation in a Hubble volume Γ/H^3 and compared it to the rate of expansion H. Finding the gravitational bounce solution that controls Γ in the semiclassical approximation is however not just hard but also beyond the limitations of our effective field theory approach. Indeed, the bounce would, by topology, involve regions of space where the euclidean time cycle shrinks to planckian length: here effective field theory breaks down, and, among other things, the physics that resolves the TeV brane becomes surely important. However for the case of a weak GW coupling $v_1 \ll (ML)^{3/2}$ the bounce action, which is large, is dominated by the region of field space over which the relevant gravitational mode is simply the radion. The basic reason for this is that for $v_1 \ll (ML)^{3/2}$ the critical temperature is smaller that the KK mass scale $\mu_{\rm TeV}$. Then there is a big range in field space $T < T_c \ll \mu < \mu_{\rm TeV}$ where the radion is the lightest degree of freedom. The "brane bubble" depicted in figure 1 lies mostly in this range. We find that the 3-dimensional bounce action S_3/T goes roughly like $(ML)^3 \times [(ML)^{3/2}/v_1]^{3/2}$ where the first factor comes just from the gravitational action being $\propto M^3$ while the second enhancement is due to the radion dominance we just mentioned. If the GW mechanism were associated to a sizeable deformation of the AdS metric around the TeV brane, i.e. if $v_1 \sim (ML)^{3/2}$, then there would be just one energy scale $T_c \sim \mu_{\rm TeV}$. In this case the whole problem should be reconsidered. In fact, now the contribution of the GW field to the action of the black hole solution is important and may well make it classically unstable at low enough temperature. Therefore the transition could now even be of second order. Anyway, even if the transition remained of first order, a full gravitational bounce solution, not just the one for the radion mode, would be needed.

Given a perturbative stabilization mechanism, if the Big Bang temperature exceeds the Fermi scale, then the transition to the present cosmological era poses a big constraint on the RS model parameters. In practice the models in which there is a secondary minimum at $\mu = 0$ are forced to be on the verge of being perturbatively untractable: the GW field must cause a sizeable backreaction on the metric and, more importantly, ML has to be so small that we expect no more than a handful of weakly coupled KK modes. Above these lowest modes the mode width is so large that they overlap to form a continuum.

However an interesting scenario is possible if we choose the parameters of the potential in such a way that $\mu = 0$ is not a stable minimum; this is possible when $\epsilon > 0$ by suitably correcting the TeV brane tension with respect to the basic RS value. In this case the small transition rate causes the Universe to inflate, but at sufficiently low temperature it will be destabilized towards the unique minimum. We expect that this will happen for $T \lesssim H$: in this regime the gravitational corrections to the transition rate are important [25] and we expect the fluctuations induced by the inflating background to be enough to overcome the potential barrier. Is this a viable scenario of inflation? If the expansion lasts until $T \lesssim H$ we have enough inflation to solve the usual smoothness and flatness problems, but it is hard to understand if the model gives a phenomenologically acceptable spectrum of density perturbations. If the theory were exactly conformal we would not have any perturbations, because the inflating Universe has a conformally flat metric. However, in our case conformal invariance is explicitly broken by the slowly running GW coupling: even if this non-conformal deformation becomes small in the IR, it runs sufficiently slowly that we do not expect a huge suppression of the induced perturbations. Further studies are required to understand the viability of this scenario.

The last "trivial" possibility is that the Big Bang started out with temperature at or below the Fermi scale, in which case one can consistently assume to be from the beginning in the RS minimum of the free energy. This poses no constraints on the parameters, though our theoretical control of early cosmology is not dramatically different than in other models with TeV scale extra dimensions.

The holographic point of view could also be useful to study non-standard cosmological solutions with non-thermal, high energy initial conditions. The solution described in the appendix may be a starting point for further studies in this direction.

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A. A radion driven cosmology

A general solution of the 5D Einstein equations in AdS_5 in presence of 4D branes can be easily obtained studying the brane motion in the static background metric [26]. The dynamics of the brane is fixed by Israel's junction conditions relating the discontinuity in the brane's extrinsic curvature to its energy-momentum tensor. For the RS model we have two branes moving in the bulk and we suppose their tension to be fixed to the values $\sigma_{\mathrm{Planck}} = -\sigma_{\mathrm{TeV}} = 24M^3/L$ in such a way that a static solution in possible.

It is rather interesting to see that, in absence of a stabilization mechanism, the trivial time independent solution is not the only possible. The generic bulk geometry is of the form (3.2) with the presence of an event horizon at $\rho = \rho_h$. We consider the geometry in which both branes are outside the horizon at $\rho > \rho_h$ ([8]). As the branes are situated at the orbifold fixed points, the horizon lies outside the physical space: the only difference with the trivial static solution is that the metric between the branes is different, *as if* a black hole horizon were present behind the TeV brane.

[8] A brane which is inside the black hole horizon is difficult to interpret from the holographic point of view as the dual theory lives outside the horizon and do not feel what is going on inside.

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Using Israel's junction conditions, we can obtain the position of each brane as a function of its own proper time $\rho_{\rm TeV}(\tau_{\rm TeV})$ and $\rho_{\rm Pl}(\tau_{\rm Pl})$. The two branes will obey the same equation

$$\dot{\rho}^2 - \frac{\rho_h^4/L^2}{\rho^2} = 0\,, \tag{A.1}$$

where ρ stands either for $\rho_{\rm TeV}$ or for $\rho_{\rm Pl}$ and the derivative is taken with respect to the brane proper time. The metric induced on the brane will have the form

$$ds^2_{\rm brane} = -d\tau^2 + \rho^2_{\rm brane}(\tau)dx^i dx^i\,, \tag{A.2}$$

so that eq. (A.1) describes the evolution of the scale factor with time. Properly fixing the origin of time, the solution of the equation of motion for the Planck brane at a sufficient distance from the horizon is

$$\rho_{\rm Pl} \simeq \sqrt{\frac{2}{L}}\rho_h \tau_{\rm Pl}^{1/2}\,. \tag{A.3}$$

The evolution of the Planck brane is the same that in absence of the TeV one and describes, in the holographic counterpart, a Universe evolving as it were radiation dominated.

At first, this behavior in presence of two branes may be puzzling. We have a "radiation dominated" Universe, but, as there is no real event horizon, the state has no entropy and is non thermal. To understand what is going on, we will study the radion evolution in the 5D picture and see that its time dependence gives, in the 4D viewpoint, the energy density and pressure which drive the Universe RD expansion.

Even if the two branes follow the same equation of motion (A.1), the radion VEV, which in the parameterization we use in the following is proportional to $\rho_{\rm TeV}/\rho_{\rm Pl}$ [12], is not constant in time. Let us calculate

$$\frac{d}{d\tau_{\rm Pl}}\frac{\rho_{\rm TeV}}{\rho_{\rm Pl}} = \frac{d\rho_{\rm TeV}}{d\tau_{\rm Pl}}\frac{1}{\rho_{\rm Pl}} - \frac{\rho_{\rm TeV}}{\rho_{\rm Pl}^2}\frac{d\rho_{\rm Pl}}{d\tau_{\rm Pl}} = \frac{\rho_h^2}{L}\frac{1}{\rho_{\rm Pl}^2}\left(1-\frac{\rho_{\rm TeV}}{\rho_{\rm Pl}}\right), \tag{A.4}$$

where we have used the relation $d\tau_{\rm Pl}/\rho_{\rm Pl} \simeq d\tau_{\rm TeV}/\rho_{\rm TeV}$, valid for $\rho_{\rm TeV} \gg \rho_h$. For $\rho_{\rm TeV} \ll \rho_{\rm Pl}$, which in the 4D language means that the radion VEV is much smaller than the Planck scale, equation (A.4) reduces to

$$\frac{d}{d\tau_{\rm Pl}}\frac{\rho_{\rm TeV}}{\rho_{\rm Pl}} = \frac{\rho_h^2}{L}\frac{1}{\rho_{\rm Pl}^2} = \frac{1}{2\tau_{\rm Pl}}\,, \tag{A.5}$$

which shows how the radion VEV evolve with the 4D scale factor and with time.

Now we switch to the 4D picture and check that this behavior can be reproduced by the 4D Einstein equations. The radion ϕ is conformally coupled to gravity ($\xi = 1/6$), its lagrangian and stress-energy tensor being [15, 16, 12]

$$\begin{aligned}\mathcal{L} &= \sqrt{-g}\left[2(ML)^3\mu_0^2 R(g) - \frac{1}{12}\phi^2 R(g) - \frac{1}{2}(\partial\phi)^2\right],\\ T_{\mu\nu} &= \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}(\partial\phi)^2 + \frac{1}{6}(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)\phi^2\,.\end{aligned} \tag{A.6}$$

The Friedmann equations for the Hubble parameter $H \equiv \dot{\rho}/\rho$ in the case of a time dependent radion are

$$\begin{aligned} H^2 &= \frac{8\pi}{3} G_4 \cdot \frac{1}{2}\dot{\phi}^2 & \text{(A.7)} \\ \dot{H} &= -4\pi G_4 \left(\frac{2}{3}\dot{\phi}^2 - \frac{1}{3}\phi\ddot{\phi} \right) , & \text{(A.8)} \end{aligned}$$

where the 4D Newton's constant is defined by $1/(16\pi G_4) = 2\mu_0^2 (ML)^3$. Using the equation of motion for ϕ,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{6} R(g)\phi = 0 \,, \tag{A.9}$$

we see that the second term in the RHS of (A.8) can be neglected in the limit $\phi \ll M_{\rm Pl}$, already used in the 5D picture. Solving for H we obtain

$$\dot{H} = -\frac{8\pi}{3} G_4 \dot{\phi}^2 = -2H^2 \, ; \tag{A.10}$$

the scale factor evolves as in a RD Universe,

$$\rho(\tau) \propto \tau^{1/2} \,, \tag{A.11}$$

consistently with (A.3). The ratio $\rho_{\rm TeV}/\rho_{\rm Pl}$ can be written in the 4D variables as $\phi\sqrt{4\pi G_4/3}$; its evolution is given by (A.7),

$$\sqrt{\frac{4\pi G_4}{3}}\dot{\phi} = H = \frac{1}{2\tau} \,, \tag{A.12}$$

which is exactly the same result obtained in the 5D picture (A.5). In the above discussion we have implicitly assumed a radion increasing with time, but a completely equivalent cosmological expansion can be obtained also with a decreasing radion, which corresponds to the other root of (A.1).

We have checked that the general 5D solution consisting of a slice of AdS-S between the two branes, in absence of a stabilization mechanism, can be interpreted in the 4D picture as a radion driven cosmology. The radion field, being conformally coupled to gravity, has a radiation-like behavior as source of gravity thus giving a RD-like cosmological evolution.

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